## P314

# Inversion of Refraction Traveltimes by Homogeneous Function Method Is Analogue of CDP Reflection Method 

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## SUMMARY

Presented below is a method of the inversion of refraction seismic data. The presentation compares interpretation of refraction data by homogeneous function method with CDP reflection depth section for the same profiles.
We show that the method of homogeneous functions used for traveltime curve inversion is an extension of the 1D classical techniques to 2D media. It is also an expansion of the CDP reflection method to refraction data. We mathematically proved that the solution of the kinematical inverse seismic problem within the set of 2D homogeneous continuous functions, which increase with polar angle, is stable.
Having used field examples, we point the applicability of the homogeneous function method for automatic reconstruction of geological structures of different geological scale and for different system observations. The internal structures in layers and faults are seen in refraction cross sections by homogeneous function method. Coincidence in detail of refraction cross section by homogeneous function method with structures in CDP reflection sections for the same profiles verifies of refraction sections reliability

## Introduction

The purpose of this research is to show the new results that can be obtained using a homogeneous function method for the inversion of refraction first arrivals in comparison with CDP reflection method.
There are some known difficulties in the inversion of refraction traveltime curves by modelling and tomography methods. Only starting model corrections can be computed by these methods. The starting model should be obtained from others sources. These corrections must be small. And thus the result cannot considerably differ from starting model. So the modelling and tomography methods cannot be considered as rigorous as independent inversions.
The modelling method requires that each segment of the traveltime curve of first arrivals is identified with some layer or seismic boundary. This is a very difficult and ambiguous problem for interpreters. These methods have ill-posed mathematical problems in their basis. Homogeneous function method, solving the same problem, is free from indicated difficulties in many relations.

## The inversion method

The homogeneous function method (Piip, 2001) automatically inverts first-arrival refractions to derive a 2 D velocity distribution which includes seismic interfaces (velocity discontinuous). By now considerable experience was gained in application of this technique for deep investigation, exploration works and for problems of shallow seismics.
We use 2D continuous homogeneous functions that increase with polar angle for fitting to real geological media. When expressed in the polar coordinates, the fitting function has the form

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\begin{equation*}
v=(r)^{m} \psi(\varphi) \tag{1}
\end{equation*}
$$

where the real number $m$ is the degree of the homogeneous function, $\psi(\varphi)$ is an arbitrary continuous function that increase with the polar angle, and $r>0$.
Functions (1) well correspond to real geological media. Similarity of geological interface is dominant property of real geological sections: synclines, anticlines, other folds and so on. Velocity contours of 2 D homogeneous functions are curves similar to one another, while the shape of the curves may be arbitrary. Homogeneous functions constitute a broad class of infinite-dimensional functions. Continuous 2D homogeneous functions that increase with a


Figure 1 View of seismic rays for two reverse traveltime curves. Turning points of the rays are shown. polar angle can vary monotonically in both vertical and horizontal directions. The values of the vertical and horizontal components of the velocity gradient have no restrictions. Velocity function (1), which increases with polar angle, can include inclined rectilinear seismic boundaries. Media with homogeneous function of velocity include as particular case multilayer media and 1D media with vertical velocity function. Minimal shooting geometry for calculating the 2D homogeneous velocity function is a pair of reversal traveltime curves; maximal geometry is not limited. Thus this method is available for interpretation of engineering seismic data (Field example 1), deep seismic sounding data, refraction first arrivals of CDP method (Field example 2 and 3) and reinterpretation of the data of past years.
The method is an extension of the classical techniques in use for traveltime simple inversion to $2 D$-varying media. Until present time we had only three methods which can be used for direct calculation (not forward modelling) of velocity values. These are the plus-minus method for refracted waves, as well as the 1D method which inverts the traveltimes of a refracted wave, i.e. the Herglotz-Wiechert (HW) formula. The 2D homogeneous function
method is an addition to the list of methods of the direct calculation of seismic velocities from traveltime curves.
Algorithm computing of $2 D$ homogeneous function from two reversal traveltime curves. We use function (1) for local fitting to real velocity section. Function (1) is computed from two reversal traveltime curves in a closed volume, outlined by a ray connecting two sources.
We find such function by a method of quasisolvings (Ivanov, 1963). In this method, we fit traveltime curves, which correspond to 2D homogeneous function, to the observed pair of traveltime curves in the best way. We proved that fitting traveltime curve is unique. After that we find the function (1) by simple inversion which has stable solution because the set of function (1) is a closed compact set. It is not necessary to use prior information and to identify any waves in observed first arrivals.
The algorithm of inversion by 2D homogeneous functions (HF) method is analogous to the Herglotz-Wiehert (HW) inversion, although the analogy is not absolute. The HF method uses functions (1), which increase with polar angle. In this case, the 2 D inverse problem can be transformed into a 1D problem (Piip, 2001). The 1D traveltime curve, obtained in this transformation, must be convex analogously to the HW method. Ray traces are simultaneously computed. Assuming that the function of the polar angle is continuously increasing, this family of rays is a regular, i.e., close-lying rays that do not intersect (Piip, 2001). The rays for two reversed traveltime curves fill the entire region confined by the ray that passes through the shot points. The turning points are located as shown in Figure 1, converging in the lower part of the velocity field. This means that the velocities can be determined with the best accuracy in this part. The computed velocity function in essence is the RMS 2D velocity function.
Construction of common section from 2D homogeneous functions can be considered as analogy to construction of Common Deep Point section in reflection seismics. If we have a set of observed refraction traveltime curves, we fit the homogeneous function (1) to actual velocity section for each pair of reversed traveltime curves selected from the entire set of curves. This procedure for every pair is independent.
Determination of the combined velocity field for a complex shooting geometry is derived by superposition of the velocity functions calculated for different pairs of traveltime curves, as shown in Figure 2. The result of this superposition takes into account only the lower (those best determined) parts of the local velocity field. The lower parts of the local velocity field can be named by a common deep zone analogously to a common deep point in reflection seismic, because the rays are focused there.
The analogy with the CDP method is not absolute. The constructions of velocity field are fulfilled in our case in the depth plane; we use 2D velocity functions and thus migration is


Figure 2 Construction of common cross section by homogeneous function method (in bottom) in comprising of construction of CDP time section (ahove) produced automatically. This is shown in Figure 2, where the offset of common deep zone from spread centre (red line) is visible. Our final section is 2D depth velocity field including seismic interfaces and thus there is no need to calculate a velocity macro model, as it is usually produced in CDP reflection seismics.
Application of the homogeneous function method to refraction data acquired with high channel counts, small receiver intervals, and high fold in area with salt tectonics showed high efficiency of the method (Piip et al, 2007).
Velocities of different local velocity fields differ a little bit from each other in overlapping areas. This is due to the fact that the inverse problem within the set of homogeneous functions is stable. The velocity deviation at a point of the section based on different pairs of traveltime curves characterizes the velocity
uncertainty, because local velocity fields are computed independently of one another.
The final velocity section is visualized with velocity values at points of a rectangular grid (the grid representation). Choosing grid size we just specify the form of the visualization of set of functions (1) which has been computed earlier. When visualizing a velocity section, the grid parameters cannot be chosen in an arbitrary manner. One would see the boundaries of local velocity fields when using a very small grid size, while a very large grid size would not allow discontinuities and other features of the velocity field to be seen at all. It thus appears that the grid size specifies the resolution available in a section. Various transformations of the final velocity field (gradient, gradient components etc.) can be used to identify features in the depth distribution of velocity.


Figure 3 Traveltime curves and cross section obtained in permafrost region.
multilayered sections can be described by homogeneous functions, and conventional geometry for engineering survey can be used.

## Field example 2

Traveltime curves and cross sections, shown in Figure 4, are obtained along regional 1-EV profile, survey along which Spetzgeofizika (Moscow) carries out in the present time. The profile, by length of more then 3000 km , crosses the central part of the European Russia from Barents Sea to Caspian Sea. Large investigations with the long offset (up 10 km ) CDP method are applied to study deep crust structures down to mantle. The first arrivals of refractions, obtained along whole profile from CDP survey, are processed by homogeneous function method. Significant complementary information was been obtained about structures in the upper part of cross section. In Figure 4 refraction traveltimes curves, CDP time section and

## Field example 1

Traveltime curves (Figure 3) were obtained with aim of study of permafrost in Pechora region (authors are A.G. Skvortzov and I.A. Galin). These traveltime curves were inverted by homogeneous function method. Any starting model was not used. Preliminary identification of waves also was not produced. The cross section in Figure 3 is grid by size $70 \times 101$ nodes. Velocity contours are showed with interval of 200 $\mathrm{m} / \mathrm{s}$. Three layered cross section, including water layer, was computed automatically. This example demonstrates that


Figure 4 Refraction traveltime curves (above), CDP time section (in middle) and depth velocity section (in bottom) for the interval of 1-EV nrofile.
depth velocity section by homogeneous function method are shown. In this interval the profile crosses the Pricaspian depression in the region of Volgograd. Sharp outlines and the inner velocity structures of the salt domes are seen in the velocity section, while location of the domes in the both sections coincides.


Figure 5 Refraction traveltime curves (above) and combined CDP depth section with depth velocity section (in bottom) fort the interval of 2-DV profile.

## Field example 3

Another regional profile, which is carried out by Spetzgeofizika, is located in the Far East of Russia. The profile, by length more then 2000 km , passes from Chukchi Sea in the North to the Sea of Okhotsk in the South. The long offset (up to 30 km ) refraction survey was fulfilled along this profile besides the long offset (10 $\mathrm{km})$ CDP seismics. This region is characterised by very complex geological structure, including folds, faults, and intrusions. Many gold and silver deposits are located in the region. Refraction traveltime curves by length of 30 km together with some traveltime curves of first arrival, obtained from CDP survey (Figure 5, upper), were processed by homogeneous function technique. Obtained velocity section (Figure 5, bottom) is combined with CDP depth section for the same interval of the profile. The geological age of the layers is indicated on the data of geological map, that is was obtained independently. This example demonstrates well coincidence CDP depth and velocity section from homogeneous function method. And also undoubtedly the received additional information has high importance for geological interpretation.

## Conclusion

1. The 2 D homogeneous functions well respond real geological sections.
2. The method is an addition to the list of known methods of the direct calculation of seismic velocities from refraction traveltime curves.
3. The simple kinematical inversion within 2 D homogeneous function set is stable.
4. The method is an extension of Herglotz-Wiehert method of the refraction inversion to 2D inhomogeneous media.
5. Local reconstruction of real geological sections by 2D homogeneous function method is analogy of CDP reflection method.

## References

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