P-152 REFRACTION TRAVELTIME INVERSION FOR 2D VELOCITY STRUCTURE USING HOMOGENEOS FUNCTIONS

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Summary

A method of simple inversion of refraction traveltimes for determination of 2-D velocity and interface structure is presented that is applicable to data both of engineering seismic and deep seismic investigations. The advantage of simple inversion, as opposed to ray tracing methods is that it provides direct calculation of 2-D velocity distribution that includes information about interfaces without calculation of seismic rays at every step of the iteration process.

Introduction

The inversion method is based on a local approximation of real velocity cross section by homogeneous functions of two coordinates (Piip, 1991). Homogeneous functions are infinite-dimension functions. In polar coordinates they are described by the product of two functions: power function of any degree from the radius and arbitrary function from the polar angle.

$$v = r^m \psi(\varphi), \qquad (1)$$

(m is the degree of homogeneous function). Thus, an infinite number of coefficients describes the homogeneous function in the case of a Taylor-series. Methods using ray-tracing are based also on local approximation of the real velocity distribution. They are used different functions of finite dimension for the local approximations. For example, the dimension of velocity function used by Zelt and Smith (1992) is 7. Methods of ray-tracing and tomography demand an initial model. Often the final velocity cross section depends on the chosen initial model. For the inversion with homogeneous functions an initial model is not needed. Preliminary distinguishing and identification of waves on traveltime curves from different sources also is not needed for this inversion method. It is produced automatically. Methods of tomography require a very detailed system of observation. Contrarily to seismic experiments that consist of numerous shots along a profile it may be impossible through trial-and-error modeling to construct a model that fits the data within acceptable limits. The method of inversion using the homogeneous function approximation is applicable to any set of traveltime curves, from a minimum of two reverse traveltime curves to a maximum when receivers and sources are located along the profile with equal spacing.

Theory

Homogeneous functions are very suitable for the approximation of real geological media because they correspond to properties of real seismic media. We know that values of the horizontal components of real velocity gradients are much smaller than values of the vertical component. The structure of homogeneous functions corresponds to this. Homogeneous functions in the radial direction change as a power function (that is a smooth function) and in the subvertical direction (depending on polar angle), they can change arbitrarily and can have discontinuities (seismic boundaries). Contour lines of homogeneous functions are arbitrary curves, however, they are similar to each other. It allows us to approximate layered geological media by homogeneous functions sufficiently well. Two main known interpretative models, a two-layer model with constant velocities and a 1-D inhomogeneous model, are special cases of homogeneous function classes. Using a local approximation by homogeneous functions one can describe an arbitrary velocity distribution.

Investigation of the eiconal equation showed that traveltime curves for media (1) have following very important properties.

1) Traveltime curves of all kinds (first arrivals, head, reflected) at the surface of a medium with a homogeneous velocity function are similar to each other. We use polar coordinates to describe the traveltime curves. For any two traveltime curves $t_1(r_1)$ and $t_2(r_2)$ with sources in points r_{01} and r_{02} on the surface $\varphi = 0$ of media (1), the following equations are fulfilled. The times, t_1 and t_2, at the points r_1 and r_2 if $r_1/r_2=r_{01}/r_{02}$ relate as $t_1/t_2=(r_{01}/r_{02})^{1-m}$.

2) Two reverse traveltime curves $t_1(r_1)$ and $t_2(r_2)$ given at the interval $[r_{01}, r_{02}]$ at the surface $\varphi=0$ of media (1) with sources at points r_{01} and r_{02} tie by nonlinear continuous transformations. The times t_1 and t_2 at points r_1 and r_2 for that product of radii is constant and equates to the product of radii of the sources $r_1*r_2=r_{01}*r_{02}$, related as $t_1/t_2=(r_{02}/r_1)^{1-m}$. These transformations map a forward traveltime curve onto an inverse one and vice versa. Function (1) can include velocity discontinuities and intervals where there is a minimum. In accordance with this, seismic media with velocity (1) can contain boundaries and wave-guides. Traveltime curves at the surface of media (1) in turn can have multi-valuednesses and shadow zones. All these singularities agree with the results of the transformations. It allows us to automatically identify waves on reverse traveltime curves. This problem is very difficult in common cases.

3) The examined media are 2D inhomogeneous. The horizontal component of velocity gradient is not value limited. However, traveltime curves for media (1) can be converted to traveltime curves corresponding to 1D media with velocity depending only on polar angle. In the special case of m=1 for function (1) transformations of traveltime curves to ones corresponding to medium with velocity depending only on depth exist.

Mentioned properties of traveltime curves allow us to turn direct and inverse problems for media (1) into 1D cases. As result of these transformations it is true that many from known inversion methods: plus - minus, formulae of Herglotz -Wiechert and others, can be used in the case of media with unlimited horizontal component of gradient of velocity.

Inverse problem

Homogeneous velocity function (1) can be calculated from two reverse traveltime curves of first arrivals. We find a best approximation of unknown velocity distribution in the media by a homogeneous function in two coordinates. The transformations of a forward taveltime curve into a reverse one (property 2) are used to find parameters of approximating homogeneous function (1): the degree *m* and *c*, parameter defining a location of polar origin at the profile. The values of *m* and *c* that minimize of mean square deviation $\sigma(c,m)$ of the given forward traveltime curve from the transformed reverse traveltime curve, are parameters of unknown homogeneous function. For minimizing of function $\sigma(c,m)$, we use standard algorithms. We note that at each step of the calculation of the values of function $\sigma(c,m)$ we do not calculate seismic rays as is done in algorithms of ray tracing, but instead we produce simple transformations of reverse traveltime curve in a forward one.

After we use the property 3 of traveltime curves –mentioned above- that allows mapping of traveltime curves onto those for 1D media. With the aim to obtain velocity boundaries in the cross section we represent unknown 1-D velocity function of polar angle as a step function with constant values of velocities in layers. In such media the head waves exist. The problem of calculating 1-D function of polar angle is solved using a formula known for head wave kinematics. Simultaneously the set of seismic rays is calculated. However in addition we must

assume that velocity values increase with increasing of polar angle, as it is demanded in kinematics of head waves. The calculated homogeneous function (1) approximates a real velocity distribution only in the area where the seismic rays, corresponding to a given pair of reverse traveltime curves, penetrate. To restrict this area we use the last ray from a set rays calculated in the process of finding velocity function - the boundary ray. The area limited by the boundary ray and the piece of surface with known velocity function, is called the local velocity field. For a complex set of observed traveltime curves the final seismic cross section is constructed from the local velocity fields.
It is shown (Piip, 1991) that the algorithm is stable and that the velocity values corresponding to different local velocity fields are well merged in the final cross section and that boundaries of local velocity fields are not apparent in the final velocity field. Numerous tests of cross sections calculated by these methods shows that they fit the data within conventional limits (for example Piip, Efimova, 1996).

Examples

The traveltime curves computed for a model with sine-shape velocity contours are represented in Fig. 1*a*. Approximately 1 % noise with rms deviation of 0.002 s was added to the computed time values. The restored model is shown in Fig. 1*b*, where the given velocity contours are represented by dashed lines and they are averaging the reconstructed ones. We note that 46 local velocity fields were used for constructing the final cross section.
Due to low demands for the shooting geometry the inversion method is used for reinterpretation of refraction data of past years. Fig.2 demonstrates possibilities of such reinterpretation on example of seismic data obtained in the Black Sea (Goncharov *et al*., 1972).

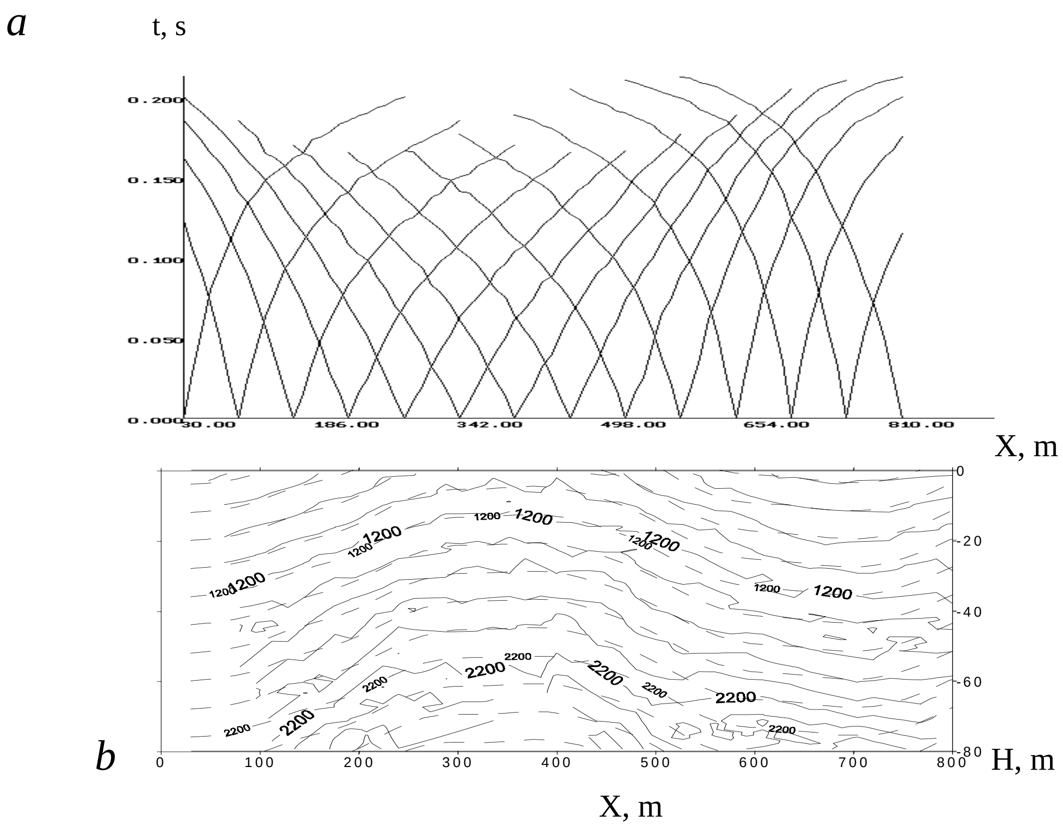
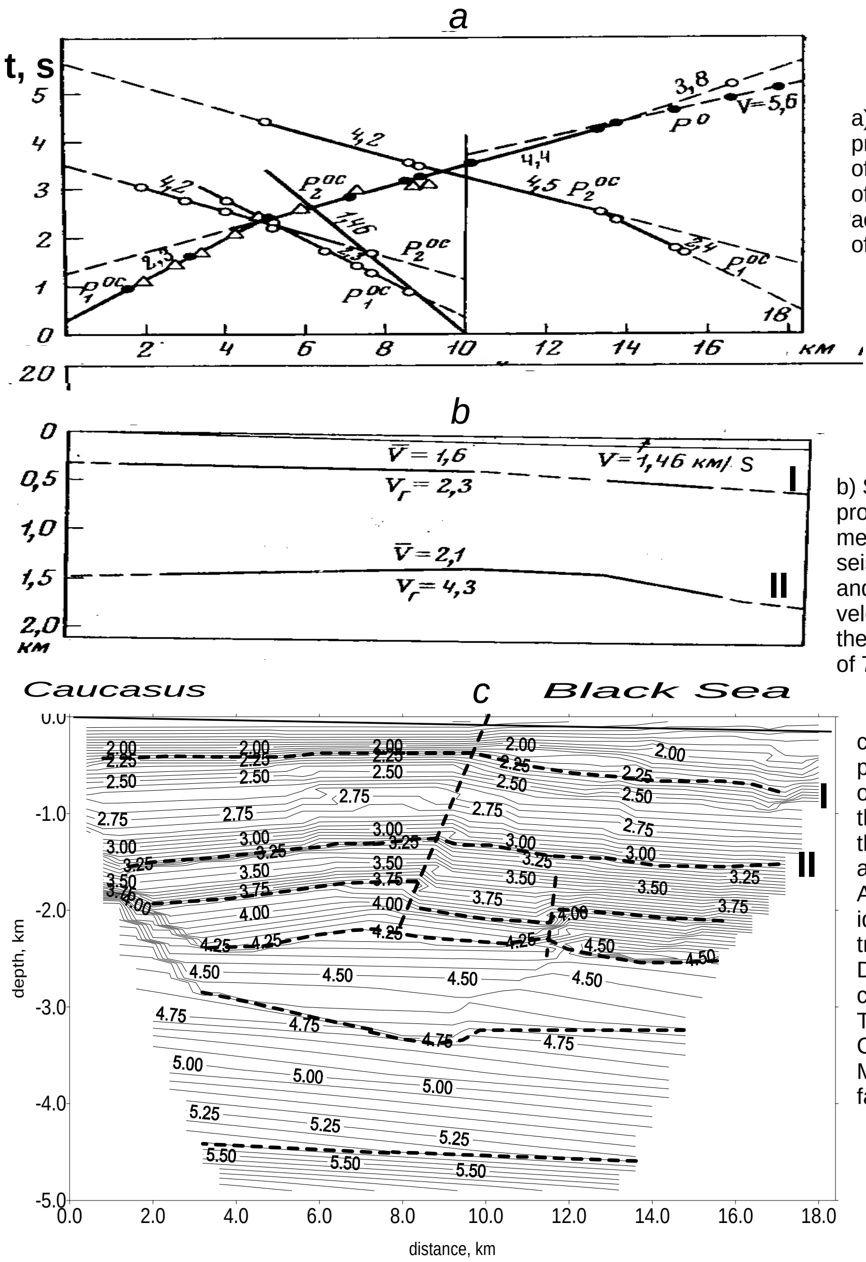


Fig. 2. Refraction traveltime curves computed for the model with sine-shaped velocity contours given with error about 1 % (*a*), (*b*) is the comparison of the given contours (dashed lines) with the result of the inversion contours (continuous lines). Contour interval is 200 m/s.



a) Traveltime curves along profile 22 with designations of head waves and values of apparent velocity according to interpretation of 70s years.

b) Seismic cross section along profile 22 calculated by using method of wave fronts. Only two seismic boundaries (I and II) and two values of refractor velocity (Vr) were received in the cross section at beginning of 70 s.

c) Seismic cross section along profile 22 computed by method of homogeneous functions. For the calculations the times in the points of obervations (circles and triangles in fig.2a) were used. An initial model and prelimminary identification of waves at the traveltime curves were not used. Depth of the authomatically computed cross section is 5 km. Thin lines are velocity contours. Contour interval is 0.05 km/s. Main seismic boundaries and faults are shown by dash lines.

Fig.2. Reinterpretation of traveltime curves received along profile 22 in the Black Sea (Goncharov et all 1972)

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